Algebra 2, Quarter 2, Unit 2.1

Working with Polynomial Functions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Prove polynomial identities.
- Use identities to describe numerical relationships.
- Know and apply the Binomial Theorem.
- Create equations and inequalities in one variable.
- Use equations and inequalities to solve problems.
- Create equations in two or more variables to represent relationships between quantities.
- Graph equations on coordinated axes with labels and scales.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Change the scale in graphing calculator views.

Look for and express regularity in repeated reasoning.

- Look for patterns in solving systems of equations.

Essential questions

- How are systems of equations used to model real-world situations?
- What is the purpose(s) for inequalities in real-life situations?
- Why are polynomial identities useful in describing numerical relationships?
- What is the purpose(s) of graphing systems of equations on coordinate plane?
- What is the purpose of proving identities?
- How would the Binomial Theorem be applied in real-life situations?

Written Curriculum

Common Core State Standards for Mathematical Content

Arithmetic with Polynomials and Rational Expressions A-APR

Use polynomial identities to solve problems.

A-APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

*A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

**Teacher note: Algebra 2 PAP objective (Optional for Algebra 2 Regular)*

Creating Equations★ A-CED

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

The first time students graphed points on the coordinate plane was in fifth grade. In sixth grade, students applied their understanding of arithmetic to algebraic expressions. Seventh-grade students solved real-life and mathematical problems using numerical and algebraic expressions and equations. Understanding and applying the Pythagorean Theorem was achieved during the eighth grade. Students also worked with integer exponents and understood the connection between proportional relationships, lines, and linear equations. Also in eighth grade, students used functions to model relationships between quantities. In Algebra I, students extended the properties of exponents to rational exponents. They also created equations that described number or relationships. They understood and solved equations as a process of reasoning and explained the reasoning. Students solved equations and inequalities in one variable and solved systems of equations. They also represented and solved equations and inequalities graphically. Students constructed and compared linear, quadratic, and exponential models and solved problems. They also interpreted expressions for functions in terms of the situation they model.

Current Learning

Students prove polynomial identities and use them to describe numerical relationships. They know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for positive integer n, where x and y are any number, with coefficients determined by Pascal's Triangle. Students create equations and inequalities in one variable and solve them. They also create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales.

Future Learning

In future mathematics courses, students will solve systems of equations, analyzing functions using different representations. They will build a function that models a relationship between two quantities and build new functions from existing functions. Carpenters, agriculture workers, medical assistants, and construction managers are professions that use the skills in this unit's standards.

Additional Findings

In this unit students synthesize and generalize what they have learned about a variety of function families. They extend the work with exponential functions to include solving exponential equations with logarithms.

Algebra 2, Quarter 2, Unit 2.2

Understanding Constraints with Models Involving Equations and Inequalities

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Write equations or inequalities to represent constraints.
- Determine whether solutions are viable or nonviable in the modeling context.
- Solve formulas for a particular variable.
- Determine if units are appropriate to limitations on measurements.
- Use units to understand problems.
- Interpret units in the context of the problem.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Need bullets.

Attend to precision.

- Need bullets.

Look for and make use of structure.

- Need bullets.

Essential questions

- Why are constraints necessary for models?
- How do you determine whether a solution is viable?
- When would you adjust a formula?

Written Curriculum

Common Core State Standards for Mathematical Content

Quantities★ N-Q

Reason quantitatively and use units to solve problems.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Common Core State Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students began to know number names and the count sequence. Equations for addition and subtraction were introduced in first grade, and the counting sequence was extended. In second grade, students extended addition and subtraction and began working with multiplication by working with groups of objects. Third-grade students solved problems involving the four operations and identified and explained patterns in arithmetic. During the fourth grade, students used the four operations with whole numbers to solve problems, and they generated and analyzed patterns. In fifth grade, students wrote and interpreted numerical expressions and analyzed patterns and relationships. They also used equivalent fractions as strategies for adding and subtracting fractions. Students applied and extended previous understanding of multiplication and division to multiply and divide fractions. In sixth grade, students applied and extended previous understandings of arithmetic to algebraic expressions. They reasoned about and solved one-variable equations and inequalities. Students represented and analyzed quantitative relationships between dependent and independent variables and solved real-life and mathematical problems using numerical and algebraic expressions and equations. In eighth grade, students used functions to model relationships between quantities. In Algebra I, students solved equations and inequalities in one variable and systems of equations.

Current Learning

Students use units to understand, interpret, and solve multi-step problems. Students graph, determining their own scales to display their data. Students determine a level of accuracy appropriate for the data they are reporting.

Future Learning

Students will use their understanding of these standards in fourth-year math courses and in college math courses. Students will continue to use these knowledge and skills in occupations such as construction manager, police detective, firefighter, and in rental industries.

Additional Findings

This content is difficult for students because they will need to identify appropriate types of functions to model a situation and they have to adjust the parameter to improve the model. They will need to compare models by analyzing appropriateness of fit and making judgments about the domain over which the model is a good fit.

Algebra 2, Quarter 2, Unit 2.3

Solving Quadratic Equations Using Various Methods

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve quadratic equations.
- Use the quadratic formula.
- Factor quadratic equations.
- Give complex solutions.

Mathematical practices to be integrated

Model with mathematics.

- Find factors of quadratic equations.
- Use a graphing calculator with the quadratic formula.

Attend to precision.

- Use a graphing calculator with the quadratic formula.

Look for and express regularity in repeated reasoning.

- Use the quadratic formula.
- Determine when you will have complex solutions.

Essential questions

- Why is there more than one way to solve a quadratic equation?
- What is the meaning of complex solutions?

Written Curriculum

Common Core State Standards for Mathematical Content

Reasoning with Equations and Inequalities **A-REI**

Solve equations and inequalities in one variable

A-REI.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Second-grade students represented and solved problems using addition and subtraction. During third grade, students represented and solved problems involving multiplication and division. The use of all four operations to solve problems started in fourth grade. In fifth grade, students wrote and interpreted numerical expressions, and they applied and extended previous understandings of arithmetic to algebraic expressions and reasoned about and solved one-variable equations and inequalities. Representation and analysis of quantitative relationships between dependent and independent variables began in sixth grade. In seventh grade, students solved real-life and mathematical problems using numerical and algebraic expressions and equations. Eighth-grade students analyzed and solved linear equations and pairs of simultaneous linear equations and worked with radicals. Algebra I students understood the relationship between zeros and factors of polynomials and solved equations and inequalities in one variable. Students constructed and compared linear, quadratic, and exponential models and solved problems in Algebra I.

Current Learning

Students solve quadratic equations with one variable. They do this by inspection, taking square roots, completing the square, using the quadratic formula and factoring. Students recognize when the quadratic formula gives complex solutions.

Future Learning

Student will continue to use solving quadratics in fourth-year and college math courses. They will continue to these knowledge and skills in fields such as engineering, natural sciences, nursing, psychiatry, home health, farming, ranching, and agricultural management.

Additional Findings

Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.

Algebra II, Quarter 2, Unit 2.4

Understanding Parabolas and Deriving Their Equations

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Find the axis of symmetry of a parabola.
- Construct the graph of a parabola using the minimum or maximum value and the axis of symmetry.
- Find the vertex of a parabola.
- Find the direction of opening of a parabola.
- Understand that the lead coefficient causes the parabola to dilate or contract.
- Determine the focus and directrix of a parabola.

Mathematical practices to be integrated

Model with mathematics.

- Use geometry to solve a design problem.
- Identify important quantities in practical situations and map their relationship using graphs.

Use appropriate tools strategically.

- Analyze graphs of parabolas using a graphing calculator.

Essential questions

- How does the geometric description of a parabola relate to the equation?
- What real-life situations can be graphed with parabolas?
- How do you derive the equation of a parabola?
- How do you find components of a parabola, including axis of symmetry, focus, directrix, and direction of opening?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressing Geometric Properties with Equations **G-GPE**

Translate between the geometric description and the equation for a conic section

G-GPE.2 Derive the equation of a parabola given a focus and directrix.

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In kindergarten, students identified and described shapes. They reasoned with shapes and their attributes in first through third grades. In fifth grade, students classified two-dimensional figures in categories based on their properties. Seventh-grade students learned how to solve equations; draw, construct, and describe geometrical figures; and describe relationships between them. Eighth-graders analyzed and solved linear equations. In Algebra I, students created equations that described numbers and their relationships, and they represented and solved the equations graphically.

Current Learning

Students derive the equation of a parabola given the axis of symmetry, vertex, and direction of opening. With this information, students find the directrix and focus of the parabola.

Future Learning

Students will use these standards in fourth-year and college math courses. They will graph parabolas and other conic sections. This content will be useful in carpentry, construction, architecture, and other careers.

Additional Findings

Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and they make connections between the zeros of polynomials and solutions of polynomial equations.

Algebra 2, Quarter 2, Unit 2.5

Using Systems of Equations to Solve for Points of Intersections

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Find the axis of symmetry.
- Construct a graph using the minimum or maximum value and the axis of symmetry
- Find the vertex of the parabola.
- Find the direction of opening.
- Understand that the lead coefficient causes the parabola to dilate or contract.
- Determine the focus and directrix of a parabola.

Mathematical practices to be integrated

- Use geometry to solve a design problem.
- Identify important quantities in practical situations and map their relationship using graphs.
- Analyze graphs of parabolas using a graphing calculator.

Essential questions

- How does the geometric description of a parabola relate to the equation?
- What are real-life situations that can be graphed with parabolas.

Written Curriculum

Common Core State Standards for Mathematical Content

Reasoning with Equations and Inequalities A-REI

Solve systems of equations.

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

Creating Equations★ A-CED

Create equations that describe numbers or relationships

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In kindergarten, students learned to understand addition as putting together and adding to and subtraction as taking apart and taking from. Students represented and solved problems involving addition and subtraction in second grade. In third grade, students learned to represent and solve problems involving multiplication and division.

Current Learning

Students derive the equation of a parabola given the axis of symmetry, vertex, and direction of opening. With this information students find the directrix and focus.

Future Learning

Students will use these standards in fourth-year math and college math courses. Students will graph parabolas and other conic sections. This will be useful in carpentry, construction, architecture and other careers.

Additional Findings

None listed.